A "MATHEMATICS FOR TEACHERS" COURSE FOR A NEW CONCURRENT TEACHER EDUCATION PROGRAM

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ABSTRACT

The paper deals with the development of a *Mathematics for Teachers* course for a new concurrent education program whose main aim is to connect academic and methodological training of teachers. After presenting the course design, it analyzes the results of its first two pilots.

1. INTRODUCTION

In the last two decades there has been a growing emphasis on the need to connect the academic and professional training of future teachers (Mohr, 2006). This is why the University of Toronto launched a concurrent teacher education program, where the two components go hand in hand. A course to connect the two is *Mathematics for Teachers*. There is ongoing research about efficient ways to develop teachers' mathematics knowledge for teaching, and similar courses are offered at a growing number of universities. These constitute a valuable input for the course design, but creating a course for the specific settings of the university calls for careful planning and testing.

2. THEORETICAL BACKGROUND: MATHEMATICS FOR TEACHERS

The need to connect subject specific and methodological knowledge in teacher training was identified in the 1980s (Shulman, 1987). Since then, there has been an ongoing research on the type of teaching specific subject knowledge mathematics teachers need, and on methods to develop such knowledge. Hill & Ball (2004) identify such knowledge to consist of common mathematical content knowledge, content knowledge teachers use in teaching that goes beyond the curriculum, knowledge of students' way of thinking about mathematics, and mathematical knowledge underlying successful methodological decisions. More specifically, Ball (2003) claims this knowledge to include knowing topics and ideas fundamental to the school curriculum and beyond, knowing connections between mathematical ideas, reasoning and proof, mathematical language and notation, and the applications of mathematics. Many studies emphasize the importance of developing problem-solving skills (Ryve, 2007), connection to other subjects (Porras, 2005), and knowledge about the school curriculum (Davis & Simmt, 2006). Developing teachers' knowledge from all these aspects should serve to develop their conceptual understanding of mathematics and their ability to think mathematically (Cooney, 1999). And finally, besides developing their knowledge of mathematics, it is also important to improve their attitude towards the subject (Gadanidis & Namukasa, 2005). On the whole, the aim is to form teachers' views of mathematics, which includes "beliefs about and attitudes towards mathematics, its learning and how it can be taught" (Watson & Mason, 2007, p. 206)

Concerning the question of how to reach these aims, it is widely agreed that future teachers need to experience the *doing* of mathematics, which can be best achieved by engaging them with mathematical content that is new to them (Davis & Simmt, 2006). New content may consist of ideas that enable them to have a higher understanding of curriculum topics. But it may also be looking at known content from a new perspective, for example by the use of extended analysis tasks: asking deep questions connected to a simple, possibly a school problem (Stanley & Sundström, 2007).

As teachers build their way of teaching from their own learning experience, the way they are engaged with mathematics should be similar to preferred ways of engaging any student with mathematics (Steffe, 1990). But it is also important for them to reflect on the learning experience itself, with Watson & Mason's words (2007):

> Although effective mathematical tasks for teachers share many of the features of effective mathematical tasks for learners, tasks for teachers also serve a higher-order purpose. To become an effective and professional mathematics teacher requires development of sensitivities to learners through becoming aware of one's own awarenesses (p. 208).

3. INSTITUTIONAL BACKGROUND: THE CONCURRENT TEACHER EDUCATION PROGRAM

The Concurrent Teacher Education Program (CTEP) was launched at the University of Toronto this academic year with first year students. It is a five years undergraduate program that includes two degrees, a Bachelor of Education, and a Bachelor of Arts, Science, Music or Physical Education. The education component is offered by the Faculty of Education (OISE), and the academic component by six other units of the university. Mathematics is offered at two suburban campuses, at Mississauga (UTM) and at Scarborough (UTSC). The aim of the program is strong cooperation between the educational and academic components, and there are two mathematics related courses that connect the two. *Mathematics for Teachers* is an academic course with a focus on subject knowledge, and *Curriculum, Instruction and Assessment in Mathematics* is an education course with a focus on methodology. The *Mathematics for Teacher* course is recommended for any student with a mathematics teachable, including future primary and secondary teachers ranging from mathematics minors to mathematics specialists.

4. METHOD: A CYCLICAL PROCESS

Designing the course is a cyclical process of collecting information, negotiation between the parties involved, planning and testing.

4.1. Collecting information

Besides the literature, information comes from three main sources: existing teacher training programs at the university, *Mathematics for Teacher* courses at other universities, and schools.

Teacher training programs at the university

The traditional, and now coexisting way of training teachers at the university is a one year consecutive program. Besides, both campuses in question offer a preparatory program for the consecutive program. I made class observations and interviewed instructors in all of these programs in order to be able to incorporate existing traditions into the course.

Schools

Schools provide an essential source of information both as future employers of teacher trainees, and as partners in teacher training. Hence I have observed several lessons and interviewed teachers, and I have also scheduled observations and interviews with pre-service teachers at practicum.

Mathematics for Teachers courses at other universities

There are *Mathematics for Teacher* courses at a growing number of universities. The setting and the purpose of each of these is different: most of them provide a basic mathematical knowledge for elementary teachers (e.g. Gadanidis & Namukasa, 2007), some are for future secondary teachers with a strong mathematical background (e.g. Senk & Hill, 2004), others consist of multiple courses focusing on various mathematical topics (e.g. McLeod & Huinker, 2007). In spite of differences, these courses serve as a valuable example. Hence I contacted designers and instructors of several of such courses (at Michigan State University, Simon Fraser University, Université Laval, University of Western Ontario, York University) to learn about their course design, syllabuses, materials, and experience.

4.2. Negotiating

As the course will be offered at two campuses with different mathematics programs, designing the course involves negation between five parties: two mathematics departments, two campus CTEP coordinators, and the central CTEP director.

4.3. Testing

Presently the groups closest to future students of the course are the ones attending the undergraduate preparatory teacher training programs mentioned in section 4.1. I carried out pilots in both groups, the results of which are analyzed in section 6. Besides, I use the course *Ideas of Mathematics* for testing, which I am currently teaching. This course is similar to *Mathematics for Teachers* in many ways: its aim is to teach students about how mathematics works, its relation to the world, and communicating mathematics. I can both borrow from this course and use it to test plans: materials, activities, and various assessment and peer review methods.

5. COURSE DESIGN

Below I outline the plans for the course.

5.1. Aims

The overall aim of the course is to develop students' view of mathematics as needed for teaching. In order to reach that, I identified the following goals:

- Deepen mathematical understanding
- Establish connections within mathematics
- Understand mathematical proof and the foundations of mathematics
- Understand the nature of problem-solving
- Understand the difference between conceptual understanding and procedural skills
- Learn about connections to other subjects (applications and cultural connections, including the history of mathematics)
- Understand the nature of mathematical communication and language
- Learn about how to incorporate teaching aids such as technology and manipulatives into teaching
- Learn about curriculum guidelines for mathematics
- Understand why it is important to study mathematics, and experience the beauty of the subject

5.2. Structure

I found that *Mathematics for Teachers* courses are organized in two different ways: either around mathematical, or around meta-mathematical and methodological topics. I decided to follow the first path. My main reason for this was the observation that most of the aims I had identified were curriculum aims as well, and the school curriculum is organized around mathematical topics, hence it will be easier for students to implement what they learn if they are provided with a model similar to that one. For each topic, I planned three components:

- Learn about the topic involving some related course aims
- Discuss the place and importance of the topic in the curriculum
- Reflect on the learning experience and teaching methods used, concentrating on one of the course aims in detail (methodological focus)

The first component serves to reach all of the aims related to mathematical understanding and connections, and to model teaching. The last two serve to raise future teachers' awareness of all of these aims, and to make them consider their implications for teaching.

5.3. Methods

As I explained before, effective ways of teaching future teachers is supposed to be similar to effective ways of teaching in general, both because this is also a learning experience and because it serves as a model for them. In particular, as the group in question will have a varied background in mathematics, differentiated teaching plays an important role. In fact, both setting a model for teachers and teaching a diversity of students call for using a variety of instructional methods, especially cooperative learning, discussions, investigations and project work (Stiff, Johnson, & Johnson, 1993).

5.4. Assessment

The double aim of the course, developing mathematical knowledge and reflecting on this learning experience, and the goal to model good teaching, call for a variety of assessment tools. Quizzes and final exams will serve to assess mathematical content and the understanding of teaching issues, and essay-like written assignments will assess the greater picture on both of these. A journal will serve reflection on mathematical and teaching related learning, and presentations and micro-teaching will help students put what they learned into practice, and serve as an opportunity to experience teaching. Students will also be prompted to reflect by peer review of written assignments, and peer feedback on presentations and micro-teaching.

5.5. Content

After negotiations between the two campuses, we established the prerequisites of the course to be *Calculus* and *Linear Algebra* on both campuses, and *Introduction to Proof* at UTM.

The material of these courses includes only a limited part of curriculum topics, and *Mathematics for Teachers* is supposed to make up for the rest of knowledge about these. However, as there is no time to cover all curriculum topics in one term, I decided to focus on big ideas and fewer topics in greater depth (Hsu, Kysh, Ramage, & Resek, 2007).

Below, I list core topics, describe two of them in more detail, and present lesson plans for two classes.

Core topics

I chose to include the following core topics in the material. The main topics are curriculum topics, and in brackets are other topics (including higher mathematics) that I plan to attach to these areas.

- Recreational (+ Predicate Calculus)
- Numbers (+ Abstract Algebra)
- Geometry (+ Proofs & Foundations of math)
- Algebra
- Counting & probability (+ Sets)
- Trigonometry

Some important curriculum strands, e.g functions, are not listed, as they will come up in connection to several areas. My aim was to touch on most curriculum strands, but to cover these as a whole, only focus on some of their big ideas.

Below I present two core topics, *Numbers* and *Trigonometry* in detail.

Sample topic: Numbers

This topic will include the following activities.

- History of mathematics: different number systems (student presentation)
- Conference on Real Numbers (Pósa, 2007) (discovery based activity – see lesson plan)
    - Properties of different number sets
    - Axioms for real numbers and simple proofs
    - Different representation of real numbers
- History of mathematics: Greek notion of commensurability and axiomatization in the 19th century (student presentation)
- Methodological focus: cross-curricular connections (cooperative activity – see lesson plan)
- Basic notions of abstract algebra: operations and algebraic structures in other topics (investigation). This open-ended activity will suit students with different mathematical backgrounds: some can investigate operations, but some can examine quite complex structures.

Sample topic: Trigonometry

- Definitions and properties: focus on why (class discussion and individual work)
- Trigonometry in real life (cooperative activity)
- Trigonometric functions: properties and transformations: focus on why (using computer software)
- Methodological focus: using technology (class discussion)
- Mathematics and music (sound waves and scales – student presentation)

Conference on Real Numbers – lesson plan

The following two-hour lesson is an adaptation of a series of activities by Pósa (2007). The aim of the activity is to develop prospective teachers' number sense, and to introduce them or deepen their understanding of the axiomatized nature of mathematics.

Introductory discussion

If students asked you "What are real numbers?" what would you answer?

Introduction to the conference

Creatures from different planets with different number concepts arrive at a conference. What kind of worlds can you think of? Suggest a set with two operations.

Besides accepting students' suggestions I trigger or suggest some of my own, so in the end we have a variety of number concepts including vectors, finite fields (under a simpler name), and the cheating addition (where $x +_c y = x + y + 1$).

Conference reception

At the conference reception creatures chat about their numbers and one of their operations. They don't have a common mathematical vocabulary, so they can only say things like: we have a number such that if we "add" it to any number we get the number we added to it. Note that "add" can mean any operation here.

- Demonstration: the two halves of the class are assigned two different worlds, and they have to discuss their numbers
- Activity in groups of four: each group is assigned two worlds, and they have to investigate similarities and differences between the two.

  Filler question: how would you define the cheater multiplication?

Plenary discussion

The aim of the discussion is to come up with rules the real numbers have to satisfy. If someone finds their numbers don't fit any more, they sadly go home. Our goal is that only *creatures with the real numbers would remain.*

Students come up with different rules, they usually get to the axioms of fields.

Theorems

Students (in groups) need to decide and prove if some simple theorems are true, false or undecidable.

Axioms for Real Numbers

At the previous activity it turned out that we cannot decide if $0 \neq 7$ holds, as it is true for finite fields, who are still in the room. So the instructor presents the rest of the axioms.

Follow-up discussion

Discussing other mathematically valid and illustrative models for the real numbers, and possibilities to answer the introductory question "What are real numbers" to the student. Discussing the axiomatized nature of mathematics.

Homework

Proving some simple theorems based on the axioms.

Cross-curricular connections – lesson plan

The aim of the lesson is to make students realize the benefit of cross-curricular activities, and to offer them examples on how to connect mathematics to other subjects.

Theoretical background

Students brainstorm the advantages of cross-curricular connections, and the instructor provides them with the basic theoretical background.

Poster preparation

Students are offered a selection of subjects other than mathematics, and they are organised into groups of 4-5 based on their subject of interest. Each group prepares a poster on topics connecting the subject to mathematics, and possible activities to exploit these.

Poster sharing

Students share their posters in four rounds. One round looks like the following: for each poster, one of the authors stays with that poster, the others go to posters that interest them. Each author presents the poster to students who joined them. There can be slight variations to this method depending on the number of students.

6. PILOTS

There have been two pilot classes, one on each campus. The classes took place at the preparatory teacher training programs described in section 5.1.

6.1. UTSC Pilot

Research question

One of the main methods of the course is engaging students with new mathematical content, however I was concerned how to do this with a group of students with diverse mathematical backgrounds. Hence my question at this pilot was to find out how an activity with strong mathematical content would fit the group, and how the teaching methods I planned would work out.

Settings

The class took place at a seminar of the Early Teacher Project for science at UTSC. I expected up to 65 students with varied mathematical backgrounds.

Method

I chose to deliver the activity *Conference on Real Numbers* described in section 5.5. At the end of the class I asked students to fill out a questionnaire. I also had a follow-up discussion with the class instructor, who is also the UTSC coordinator of the Early Teacher Project and of CTEP.

Results and Discussion

<u>My observations</u>

During the class, I experienced mixed reactions from students. The group was very active in class discussions. During groupwork, I found that the majority of the groups were greatly involved, but a few of them became passive as the difficulty of the problems increased. On the other hand, as I expected, there were students who had obviously heard about the axioms already. This was also a concern to me, as the aim of the activity was to discover the axioms.

The other issue I was concerned with was timing. As students were worried about some practical issues, part of the class was spent resolving these, so I had less time than what I had planned for. So we did not have enough time for the *follow-up discussion*, which left me with the feeling that I could not clearly communicate the aim of the class to students.

<u>Student feedback</u>

I received 43 questionnaires. The questionnaire asked about students' backgrounds and aims first, then asked them 5 multiple-choice questions about the difficulty of the mathematics involved and the usefulness of the lesson for them as future teachers, then asked them to comment. 36 students claimed they plan to teach mathematics, and I only analyzed their questionnaires in detail. Among these, I identified three main groups:

Understood the mathematics well, and found the lesson useful. 70% of students fell into this group. Many of them commented on the interesting content, and the usefulness of thinking about numbers in new ways. They also commented on the usefulness of group activities and interactive discussions. One student wrote that the class was more useful as a university lecture than a seminar that prepared them for teaching.

Understood the mathematics well, but did not find the lesson useful. 25% of students fell into this group. Two students commented that the material was too basic for them, and two wrote that it was too difficult for a school activity. One student thought that the objective was not clear, and another that he/she would have preferred to learn about teaching techniques. Two of them praised the communication and the presentation.

Did not understand the mathematics well, and did not find the lesson useful. 5% of students fell into this group. Both of them complained about not understanding the material.

Regarding the level of mathematics involved, the results suggest that it was appropriate for most students, as only two of them wrote it was too easy, and two that it was too difficult. Some also emphasized that it was interesting. Several students praised the teaching methods used, especially group activities and the interactivity in the discussions. Regarding usefulness for teaching, it is clear from the comment that I did not communicate the aim of the lesson well. A student wrote this explicitly, and several comments about this not being an appropriate school activity, or about not discussing teaching techniques made this clear.

<u>Feedback from the instructor</u>

The instructor observed the same mixture of student involvement I had. He added that students less involved were the ones that are not planning to teach mathematics. This coincides with the results of the questionnaires. He also claimed that he did not see it as a problem that some students have heard about the axioms already, as it offered them a possibility to deepen their understanding.

Conclusion

To sum up the results, the mathematical content of the task was appropriate for most students, although they had different backgrounds in the area. This was possible because of the teaching methods used, especially groupwork. At the same time, some students did not understand the use of the activity for them as future teachers. This was partly due to insufficient communication because of time constraints. But it can also be accounted for by Watson & Mason's (2007) observation that tasks designed so that teachers can experience learning on their own levels are sometimes rejected as not relevant to their teaching.

As an implication for the course, teaching strong mathematical topics with methods that allow for differentiation will be appropriate, and proper time will have to be allowed for reflection and methodological discussion connected to these activities.

6.2. UTM Pilot

Research question

Besides learning about mathematics, the other main aim of the course is to reflect on the learning experience and relate it to methodology. My question was if the content and methods I planned for such activities, called *methodological focus*, would be appropriate.

Settings

I carried out the pilot at the Science Education Seminar of UTM. There are 36 students enrolled in the course with various subjects, in fact only 2 of them are mathematics students. I had 50 minutes for the activity.

Method

I chose to try the activity *Cross-curricular connections* described in section 5.5. As students came from a wide range of programs, and only 2 from Mathematics, I slightly modified the activity. Instead of asking different groups to look at connections between mathematics and a specific subject, every group had to take a different subject and look for connections to all other subjects. Because of time constraints, I only planned for two rounds of presentations in the end, and planned to spend less time on each phase that I normally would. At the end of the class I asked students to fill out a questionnaire.

Results and Discussion

<u>My observations</u>

All students were very active during every phase of the lesson. They came up with very good ideas for the posters. Afterwards, they presented their ideas well, and in most cases the audience joined in for discussion in the end. I was very strict about timing and spent less time on each activity that I normally would, but I had the impression that this made the lesson more action-packed.

<u>Student feedback</u>

The questionnaires had four open questions. The first two asked about the importance of the topic, and about what they learned and if it was useful for them as future teachers. All 26 students present thought the topic important and what they learned useful. Many students used superlative words such as ‘extremely’, and they wrote enthusiastically about what they learned and why the topic was important. The third question was what they liked about the seminar. Most students gave their opinion on the teaching methods used, many of them liked the interactivity, hands-on approach, groupwork, engagement, organization, and that it was a different experience. Some students mentioned which activities they liked: mostly making and presenting posters, only two referred to the theoretical discussion. The fourth question asked for suggestions. Ten suggestions asked for more time, either for preparing posters, or in order to view more presentations. Two students asked for more theory, three for a powerpoint or visuals accompanying the theory, and two for clearer instructions.

Feedback from the instructor

My impressions and student questionnaires gave me a quite clear picture of the results, so I did not feel the need to schedule a follow-up interview with the instructor. In our informal discussion after the lesson she said she thought the activity worked very well, and later she wrote me that "the class really enjoyed the activity and they thought it was an excellent way to get students thinking about the connections between subjects".

Conclusion

On the whole, students were very motivated by the topic, and they also found teaching methods effective. Student feedback also suggested that I do need to spend a little more time on each activity in the future. This suggests that *methodological focus* will be a valuable and rewarding component of the course.

7. NEXT STEPS

The pilots suggested that the kind of content and methods planned were appropriate, and they also implied some issues to pay attention to. So I need to carry on planning the course in the same spirit. There will be a full pilot of the course in the near future, during and after which I will be able to make further adjustments.

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